'Some' and 'exists'

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In this paper I wish to examine certain specific connections between PM-type logic and 1) the doctrine of existential import, 2) the question of whether 'exists' can be a predicate, 3) the problem of naming, and 4) Russell's theory of descriptions. The several points I make, I think, will suggest that analytic philosophy could, without loss, dispense with the doctrine of existential import, and that if it did so a variety of inconsistencies, queer dicta, and needless controversies would be eliminated. But in this paper, such a thesis must appear merely as a strong suggestion. I shall only attempt to demonstrate its credibility with respect to a few portions of the material that must eventually be considered.

I

By the "doctrine of existential import" we refer to various formulations of the view that every statement of the form "Some x is F" is equivalent to a statement of the form "There exists at least one x that is F," or some similar expression asserting existence. The doctrine appears in modern logic as far back as Leibniz[1] and is implicitly asserted in Principia

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[1] Cf. Lewis, C. I. A Survey of Symbolic Logic, 1918, pp 14-15. Lewis gives Gerhardt's Philosophischen Schriften von Leibniz, Berlin, 1890, vii, pp 212-214, as sources.

Mathematica when the existential quantifier is introduced:

> "We shall denote 'ϕx sometimes' by the notation $(\exists x).\phi x$. Here '$\exists$' stands for 'there exists,' and the whole symbol may be read, 'there exists an x such that'."[2]

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[2] Whitehead, A. N. and Russell, B., Principia Mathematica, (1st ed) 1913, Vol. 1, p 127. Note: Section *9, from which this quotation was taken, was replaced by *8 in the second edition (Cf. xxv, and App. A. in 2nd ed. 1927). Although *8 omits this phrasing, there is no attempt to repudiate it.

The common practice in logic textbooks since has been to provide in effect two or more semantic rules for the same symbol, one of which involves "some" and one of which involves "exists." This, and indeed the practice of calling the symbol involved the '<u>existential</u>' quantifier, suggests how thoroughly the doctrine of existential import is embedded in contemporary logic. Nevertheless, we can, and will, ask whether this doctrine is an <u>essential</u> element in mathematical logic, or a gratuitous and dispensable addendum.

To make clear <u>what</u> we are thinking about eliminating, the following distinctions must be made.

a. We shall not consider eliminating anything which would result in restrictions on the syntactical transformations of PM logic. Syntactical definitions or abbreviations, like Quine's

D8. $\ulcorner(\exists\alpha)\urcorner$ for $\ulcorner\sim(\alpha)\sim\urcorner$,

for example, or syntactical systems leading to $\vdash \ulcorner(\exists\alpha)\phi \equiv\sim(\alpha)\sim\phi\urcorner$, must not be affected by any elimination we may make. Syntactical restrictions will be treated as affecting the <u>essential</u> structure of PM logic; and thus are ruled out by our present purposes. We shall be concerned, therefore, only with the dispensability of certain semantic rules.

b. We are not willing to dispense with <u>all</u> semantic rules. A pure syntactical system can only lay claim to being a <u>formalized logic</u> if, with certain semantic rules appended, it yields a satisfactory number of statements recognizable as belonging to what is traditionally called "the principles of logic". We shall assume, with respect to quantification theory, that at least the following semantic rules for the symbol, '(∃x)', must hold if this requirement is to be met:

SR1 (Semantic Rule 1) '(∃x)' may be interpreted as "It is false that for all x, it is false that..."

$SR2_a$ '(∃x)' may be interpreted "For some x.."

or,

$SR2_b$ '(∃x)' may be interpreted "At least one x is such that"

The first of these is based on substitutions, where 'for all x' is assumed to have been previously associated with '(x)', the standard interpretation is assumed for '~', and the syntactical equivalences include those mentioned in a above. The second rule seems necessary to preserve a minimum connection with traditional logic and common meanings. Others could no doubt be added, (e/g/. based on 'whatever x may be' for '(x)') but these are not at issue here.

c. The question of the dispensability of the doctrine of existential import ammounts to asking whether it is necessary or desirable in mathematical logic to add, in addition to semantic rules like S1, S2a, and S2b, semantic rules like

SR3. '(∃x)' may be interpreted 'There exists at least one x such that', (or by similar expressions which assert existence).

When a semantic rule like SR3 is added to rules like SR2 and SR1, we shall assume that the doctrine of existential import has been implicitly asserted. And from such groups of rules, and the standard syntactical transformations, it is easily shown that various meta-semantical propositions, such as, "all universal statements are denials of existence, while all particular statements are affirmations of existence" will follow as consequences.

II

With these distinctions in mind, let us consider first what was accomplished - and what was not - in an article by Nakhnikian and Salmon several years ago on 'exists' as a predicate.[3]

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[3] Nakhnikian, G. and Salmon, W., "'Exists' as a predicate", <u>Philosophical Review</u>, Oct. 1957, pp 535-42.

These authors showed, quite conclusively, that certain arguments by Wisdom, Ayer and Broad, were mistaken.[4] The latter had argued that 'exists'

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[4] References were to John Wisdom, <u>Interpretation and Analysis</u>, (London, 1931), p. 62,; A. J. Ayer, <u>Language, Truth and Logic</u>, 2nd Ed. (London, 1947), p. 43; C. D. Broad, <u>Religion, Philosophy and Psychical Research</u>, (London, 1953), pp. 182-183.

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could not be a predicate for two reasons: to allow this would make a) all affirmative existential propositions tautologies, and b) all negative existential propositions self-contradictories. Nakhnikian and Salmon showed that such arguments rested on a) incorrect translations of affirmative existential propositions and/or b) mistaken identifications of self-contradictions. They showed further that no inconsistency would arise from allowing such statements as "Some horses exist" and "No horses exist", to be translated properly into PM logical symbolism as '$(\exists x)(Hx.Ex)$' and '$(x)(Hx \supset \sim Ex)$' respectively, and that admission of the predicate 'exists' gives no support to the ontological argument.

This was accomplished without raising any questions about the doctrine of existential import. In fact, Nakhnikian and Salmon took this doctrine for granted, as will be shown shortly; and because they took it for granted they were led to hold that '$(x)Ex$', i.e., "Everything exists", not only entails no inconsistency, but expresses a necessary truth about the predicate 'exists', namely that it is a universal predicate (cf. Op. cit. p. 540). But by the same token, they did not remove one of the strongest arguments, based on PM logic, for refusing to admit 'exists' as a predicate.

III

To see clearly what this argument is, let us examine an argument by Quine which is apparently related to quite a different conclusion. In his Mathematical Logic, Quine takes the position that primitive names, like 'Pegasus', 'God', and 'Europe' in ordinary language, should all be replaced by descriptions, of the form '($\imath$x) x is pegasus', '($\imath$x) x is god', etc., in his logically ideal language. Quine's argument is as follows:

> "Actually the suggested course has, despite such artificiality, a considerable theoretical advantage; for the following difficulty otherwise arises in connection with the notion of existence.
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> "To say that something does not exist, or that there is something which is not, is clearly a contradiction in terms; hence '(x)(x exists)' must be true. Moreover, we should certainly expect leave to put any primitive name of our language for the 'x' of any matrix '...x...', and to infer the resulting singular statement from '(x)(...x...)'; it is difficult to contemplate any alternative logical rule for reasoning with names. But this rule of inference leads from the truth '(x)(x exists)' not only to the true conclusion 'Europe exists' but also to the controversial conclusion, 'God exists' and the false conclusion 'Pegasus exists', if we accept 'Europe', 'God', and 'Pegasus' as primitive names in our language."[5]

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[5] Quine, W. V. O., Mathematical Logic, Harvard Univ. Press, 1958, p. 150.

Now the first step in Quine's argument is to establish that '(x)(x exists)' must be true. This is precisely the proposition, '(x) Ex', that Nakhnikian and Salmon said expresses the special characteristic, universality, which is peculiar to the predicate 'existence'. In their words, it "states merely... that it is not the case that there exist non-existent entities", and for extensional logic, "constitutes a complete specification of the meaning of 'exists'" (p. 538). Granted this first step, Quine goes on to show that if we permitted, by a rule a universal instantiation, the primitive names of ordinary language to be put in the place of variables when the quantifier is dropped, then by the necessary truth of '(x)(x exists)' and the rule of universal instantiation, the necessary truth of 'Pegasus exists', and 'God exists', as well as 'Europe exists' would follow. And, understandably, he held that it would be intolerable to admit, as truths derived from logic, propositions asserting the existence of fictitious or debatable entities like Pegasus and God.

Quine solves this difficulty by tolerating 'exists' as a predicate, but refusing to permit names to be substituted directly when instantiating. But this is not the only means of handling the difficulty. One might prefer, as Russell did, to permit names to be used freely in instantiating while avoiding the difficulty by eliminating the word 'exists' as a first order predicate. Note that this does not mean that ordinary language expressions like "Russell exists" are called nonsense; it means that the predicate variables in the 1st order predicate calculus, 'F', 'G', 'H', ...in wffs like '(x)(Fx $\supset$ Hx)', can not be replaced by the ordinary language predicate 'exists'. Quine avoids the difficulty mentioned by restricting the ordinary language expressions that can be substituted for individual variables (namely, by excluding proper names; Russell avoids it by restricting what can be substituted for the predicate variables in his system (namely by ruling out 'exists' as a predicate). Both Russell and Quine want to get plausible logical translations of such *ordinary* expressions as 'Russell exists', or 'The author of Waverley exists' back into their logic however, and both, though in different ways, do it by utilizing Russell's theory of descriptions.

Now the difficulty that Quine pointed out so forcefully is one that Nakhnikian and Salmon did not meet. They recognized the proposition, "Everything exists", as a statement that must be considered true if we accept PM logic, but they did not deal with the difficulty this creates: *either* we must refuse to admit 'exists' as a predicate, *or* we must accept the necessary truth of 'God exists', 'Pegasus exists', etc., *or* we must find some third way out of the consequences of this universal proposition. They did not, therefore, complete the job of showing that 'exists' can be treated as a predicate.

IV

It is not correct that *both* a) the queer position that 'exists' can not be a predicate *and* b) the queer position that proper names can not be substituted freely in instantiations from universals, must be retained as long as we retain the doctrine of existential import. But is *is* clear, from Quine's argument, that *one* of these two doctrines must be retained as long as we have a) a rule of universal instantiation, *and* b) the proposition that '(x)(x exists)' as a truth.

But it is also clear that the truth of '(x)(exists)' *need* not be accepted, and therefore *neither* of these two queer doctrines need be held, if the doctrine of existential import, i.e., SR3, is dropped. For this, and only this semantical rule makes the expression

(1) $(\exists x) - x$ exists

read as

(2) "There exists at least one x such that x does not exist"

and clearly it is this sort of *reading* that Quine had in mind when he said, "To say that *something* does not *exist*, or that there *is* something that *is not* is clearly a contradiction in terms;...". If this reading is a contradiction then its denial,

(3) $-(\exists x) - $ (x exists)

and the syntactical equivalent of this denial,

(4) (x)(x exists)

must be logically true. And it is because they used some semantical rule like SR3 that Nakhnikian and Salmon said that (4) "merely states that everything exists, or that it is not the case that there exist non-existent entities" (p. 538). The purported equivalence of the underlined expression (my underlining), i.e., of (3) above, with (4) can only rest on the doctrine of existential import, i.e., some rule like SR3.

More specifically, without existential import (SR3) statement (4) is still read "Everything exists", but there is no longer any reason to say it must be true, and this is because, without SR3 the statement (1), '$(\exists x)-$ (x exists)', no longer clearly contains a contradiction. By SR1, SR2a or SR2b it says merely

(5) "It is false that for all x, x exists"

or (6a) "For some x, x does not exist" or "Some things don't exist"

or (6b) "At least one x is such that x does not exist"

and none of these bear prima facie evidence of a contradiction; they do not, for example, like (2), contain two occurrences of 'exists', one affirmed and the other denied of the same object. Nor do they seem clearly contradictory in terms of ordinary usage. The statement, "Some things don't exist; for example, Pegasus and Santa Claus and round squares" seems to convey a significant truth in a natural and perfectly clear way.

Thus the "difficulties in the notions of existence" which Quine points to as requiring the replacement of proper names by descriptions, and which would, in the absence of that solution, provide a compelling reason for refusing to admit 'exists' as a predicate, are seen to rest at bottom, not on the word 'exists', but on the doctrine of existential import which is embedded in modern logic.

V

Finally, let us consider some interesting results with respect to the significance and uses of the theory of descriptions. Russell, as well as Quine, seemed to feel that descriptions were somehow useful in solving "difficulties in the notion of existence". It was supposed that a chief virtue of descriptions was that it provided a translation into logical symbolism of assertions of individuals' existence without using 'exists' as a predicate, or else, in Quine's case, without making proper names instantiable. With the elimination of SR3 descriptions can no longer bask in the glory of this virtue, for there is no need to avoid 'exists' as a predicate, or to replace proper names, and the existential quantifier can no longer be counted on to re-introduce the notion of existence. But this does not mean that the theory of descriptions has no virtue left at all.

Let us consider Russell's theory in more detail. Essentially he held that for the proposition "The author of Waverley exists" to be true, the propositional function "x writes Waverley" must be

a. true for at least one x

b. true for at most one x.[6]

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[6] Cf. "The Philosophy of Logical Atonism", Sect. VI, in Russell, Logic and Knowledge, 1956, esp. p. 249.

This distinguishes very neatly the components which went into Russell's theory of descriptions. Taken separately, they are partially symbolized

(7) $(\exists x)$(x writes Waverley)

(8) (y)(y writes Waverley $\equiv$ x=y)

and combined, they read

(9) $(\exists x)$(x writes Waverley. (y)(y writes Waverley $\equiv$ x = y))

Russell interprets (7), using existential import, (SR3), as "There exists at least one x, such that x writes Waverley" and (9) is likewise read "There exists at least one x such that x writes Waverley and for all y, y writes Waverley if and only if y is the same as x", or more briefly, "There exists one and only one x such that x writes Waverley", or more briefly still, "The writer of Waverley exists". When SR3 is dropped, the word 'exists' drops out. Instead, (7) reads simply "For at least one x, x writes Waverley" or "Some one wrote Waverley", and (9) reads "one and only one x writes Waverley." Neither of these readings say anything about existence.

But instead of talking about the author of Waverley, who existed, let us talk about the Greek God Hephaestus who was the only Greek God who was not physically perfect, since he had a lame foot, and who was called 'Vulcan' in Latin, although none of us would assert he existed. Now the following sentences are significant, and those who know about the Greek Gods would call them true.

(10) "Some Greek God was a lame God"
or "At least one Greek God was a lame God"
symbolized by '$(\exists x)$(x was a Greek God. x was lame)'

(11) "One and only one Greek God was a lame God", i.e.,
"At least one Greek God was a lame God and at most one Greek God was a lame God", symbolized as '$(\exists x)$(x was a Greek God . x was lame . (y)(y was a Greek God and y was lame $\equiv$ x = y))'

But (10) has essentially the same form as (7), and (11) has essentially the same form as (9). If we wish to talk about "the lame God of the Greeks", which is where descriptions enter the picture, then we can say

(12) "The lame Greek God was Hephaestus", i.e., "one and only one

Greek God was lame and that god was Hephaestus", symbolized in the manner of Descriptions by

(ιx)(x was a Greek God. x was lame) = Hephaestus

or, eliminating the abbreviation '(ιx)',

$(\exists x)$(x was a Greek God. x was lame. (y)(y was a Greek God and y was lame $\equiv$ x = y). x = Hephaestus)

or (13) "The lame Greek God was male" symbolized by

$(\exists x)$(x was a Greek God . x was lame . (y)((y was a Greek God.y was lame) $\equiv$ x = y). x was male)

The latter follows precisely the schema outlined by Russell for analysis of descriptions.

But no mention has been made of 'existence', nor do I see, without the doctrine of existential import, how it would follow. None of these statements, (10)-(13) would ordinarily be supposed to entail the statement

(14) "The lame Greek God exists"

although, with the doctrine of existential import, this would be entailed by all of them and they would all be false. If existential import were dropped it would be necessary to introduce (14) as new independent statement, different from (11). It would have the form

(15) $(\exists x)$(x is a Greek God. x is lame. (y)(y is a Greek God and y is lame $\equiv$ x = y). x exists)

And this would be false, though (10) to (13) are counted true, because Greek Gods don't exist. Its form is exactly the same as that for any other predicate, e. g., (13) above.

Such a separation of 'existence' from the device of descriptions would be beneficial in at least two ways. It would allow us to preserve the distinction between such truths as (10) - (13) and such falsehoods as "No Greek Gods were lame", "Two Greek Gods were lame" and "The Greek God that was lame was Apollo". As long as existential import is retained, we arrive at the odd position that we must count the first of these true, and (10) -(13) all false, if we agree that Greek Gods do not exist. But this is, to say the least, quite an unnatural sort of judgment. Secondly, the elimination of existential import would permit uniformity in the treatment of all predicates of ordinary language, including 'exists', vis-a-vis their formulations in logic. Thus far from robbing descriptions of their virtue, the elimination of existential import would make the true virtue of this device stand forth much more clearly - namely, its usefulness in translating ordinary predications about <u>definite described</u> (vs. named) individuals into statements which would require no primitive constants beyond those available in the 1st order predicate calculus.

While I have by no means touched upon all of the objections and arguments to which my thesis is subject, I think I have shown that there is good reason to re-examine critically the assumption that the doctrine of existential import is essential in modern logic. Such a re-examination, I believe, will reap a rich harvest of solutions to supposed difficulties and queer dicta and doctrines that have been attributed, unjustly, to "difficulties in the notion of existence".

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